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Notes on upper bounds for the largest eigenvalue based on edge-decompositions of a signed graph

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ABSTRACT

The adjacency matrix of a signed graph has $+1$ or -1 for adjacent vertices, depending on the sign of the connecting edge. According to this concept, an ordinary graph can be interpreted as a signed graph without negative edges. An edge-decomposition of a signed graph $\dot{G}$ is a partition of its edge set into (non-empty) subsets $E_1, E_2, \ldots, E_k$. Every subset E_i $(1 \leq i \leq k)$ induces a subgraph of $\dot{G}$ obtained by keeping only the edges of E_i. Accordingly, a fixed edge-decomposition induces a decomposition of $\dot{G}$ into the corresponding subgraphs. This paper establishes some upper bounds for the largest eigenvalue of the adjacency matrix of a signed graph $\dot{G}$ expressed in terms of the largest eigenvalues of subgraphs induced by edge-decompositions. A particular attention is devoted to the so-called cycle decompositions, i.e., decompositions into signed cycles. It is proved that $\dot{G}$ has a cycle decomposition if and only if it is Eulerian. The upper bounds for the largest eigenvalue in terms of the largest eigenvalues of the corresponding cycles are obtained for regular signed graphs and Hamiltonian signed graphs. These bounds are interesting since all of them can easily be computed, as the largest eigenvalue of a signed cycle is equal to 2 if the product of its edge signs is positive, while otherwise it is $2\cos\frac{\pi}{j}$, where j stands for the length. Several examples are provided. The entire paper is related to some classical results in which the same approach is applied to ordinary graphs.

1. Introduction

A signed graph $\dot{G}$ is an ordered pair (G, σ), where $G = (V, E)$ is an ordinary graph, also known as the underlying graph, and $\sigma: E \to \{-1, +1\}$ is the sign function or the signature. The edge set E of a signed graph is partitioned into subsets of positive and negative edges, denoted by E^+ and E^-, respectively. An ordinary (unsigned) graph is viewed as a signed graph with the all positive signature. The number of vertices (also known as the order) of a signed graph is denoted by n.

The adjacency matrix $A_{\dot{G}}$ of $\dot{G}$ is obtained from the adjacency matrix of G by replacing $+1$ with -1 whenever the corresponding edge is negative. The eigenvalues of $\dot{G}$ are the eigenvalues of $A_{\dot{G}}$. In particular, the largest eigenvalue, also known as the index, is denoted by λ_1, or $\lambda_1(\dot{G})$ if the corresponding signed graph needs to be specified.

An edge-decomposition of a signed graph $\dot{G}$ is a partition of its edge set $E(\dot{G}) = \{E_1, E_2, \ldots, E_k\}$. It can also be written $E(\dot{G}) = E_1 \sqcup E_2 \sqcup \cdots \sqcup E_k$. Every subset E_i $(1 \leq i \leq k)$ induces a subgraph $\dot{G}|_{E_i}$, and $\dot{G}$ is decomposed into $\dot{G}|_{E_1}, \dot{G}|_{E_2}, \ldots, \dot{G}|_{E_k}$. For more details on signed graphs, their eigenvalues and eigenspaces, the reader is referred to (Zaslavsky, 2010).

The purpose of this paper is to give some upper bounds for $\lambda_1(\dot{G})$ expressed in terms of $\lambda_1(G|_{E_i})$, $1 \leq i \leq k$. The particular case when $\dot{G}$ is decomposed into cycles and the case when $\dot{G}$ is regular are also considered. In addition, this paper deals with Eulerian and Hamiltonian signed graphs, where a signed graph is regular, Eulerian or Hamiltonian whenever the same holds for its underlying graph. Some related results can be found in the previous works (Stanić, 2018, 2019a, 2022). This study is also related to the works of Fiedler and Mohar, see (Fiedler, 1973, 1975; Mohar, 1991), where a similar approach is applied to ordinary graphs.

The entire contribution is reported in the next section. Concluding remarks are given in Section 3.

2. Results

A signed cycle is positive if the product of its edge signs is 1; otherwise, it is negative. A signed graph is balanced if it has no negative cycles. If U is a subset of the set of vertices V of $\dot{G}$, then the switched signed graph $\dot{G}^U$ is obtained by reversing the sign of every edge with one end in U and

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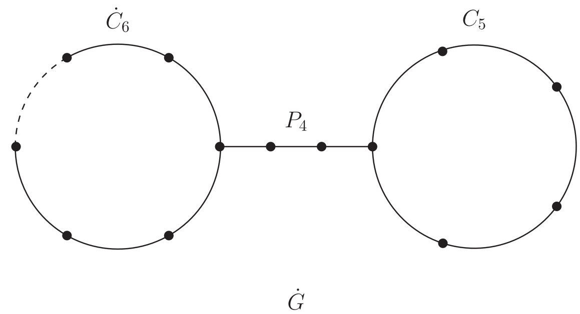

Fig. 1. The signed graph $\dot{G}$ for Example 2.1.

the other in $V \setminus U$. A signed graph is balanced if and only if it switches to its underlying graph (Zaslavsky, 2010). We know from (Stanić, 2019a, 2019b) that $\lambda_1(\dot{G}) \leq \lambda_1(G)$, along with equality if and only if $\dot{G}$ is balanced.

Two subgraphs are said to be vertex-disjoint if they have no common vertices. Let $\mathcal{D}$ denote the collection of all edge-decompositions of $\dot{G}$. For $D \in \mathcal{D}$, let V_D denote a partition $\{D_1, D_2, \ldots, D_\ell\}$ of D such that subgraphs induced by D_i $(1 \leq i \leq \ell)$ are vertex disjoint, and let $\mathcal{V}_D$ denote the collection of all such partitions.

Example 2.1. Fig. 1 illustrates the signed graph $\dot{G}$ with exactly one negative edge, depicted by a dashed line. An edge-decomposition, say D, of $\dot{G}$ induces the negative cycle $\dot{C}_6$, the positive cycle C_5 and the path P_4 lying between them. Formally, $D = \{E(\dot{C}_6), E(C_5), E(P_4)\}$. There are exactly two partitions of D such that the corresponding subgraphs are vertex-disjoint: the first is D itself and the second one is $\{E(\dot{C}_6) \cup E(C_5), E(P_4)\}$, as two cycles are vertex-disjoint.
Here is the following result.

Proposition 2.2. *It holds*

$$\lambda_1(\dot{G}) \leq \min_{D \in \mathcal{D}; \{D_1, D_2, \ldots, D_\ell\} \in \mathcal{V}_D} \sum_{i=1}^{\ell} \lambda_1(\dot{G}|_{D_i}).$$

Proof. Let D denote an arbitrary edge-decomposition of $\dot{G}$ and $\{D_1, D_2, \ldots, D_\ell\}$ denote an arbitrary partition of D. Let also A_i denote the matrix obtained from $A_{\dot{G}}$ by replacing every element outside the submatrix $A_{\dot{G}|_{D_i}}$ with a zero. Clearly, $A_{\dot{G}} = \sum_{i=1}^{\ell} A_i$. By the Rayleigh principle,

$$\begin{aligned}\lambda_1(\dot{G}) &= \max_{\mathbf{x} \in \mathbb{R}^n, \|\mathbf{x}\|=1} \mathbf{x}^\intercal A_{\dot{G}} \mathbf{x} = \max_{\mathbf{x} \in \mathbb{R}^n, \|\mathbf{x}\|=1} \mathbf{x}^\intercal \left(\sum_{i=1}^{\ell} A_i \right) \mathbf{x} \\ &= \max_{\mathbf{x} \in \mathbb{R}^n, \|\mathbf{x}\|=1} \sum_{i=1}^{\ell} \mathbf{x}^\intercal A_i \mathbf{x} \leq \sum_{i=1}^{\ell} \max_{\mathbf{x}_i \in \mathbb{R}^n, \|\mathbf{x}_i\|=1} \mathbf{x}_i^\intercal A_i \mathbf{x}_i.\end{aligned} \tag{1}$$

If the order of $\dot{G}|_{D_i}$ is n_i, then it holds

$$\max_{\mathbf{x}_i \in \mathbb{R}^n, \|\mathbf{x}_i\|=1} \mathbf{x}_i^\intercal A_i \mathbf{x}_i = \max_{\mathbf{y}_i \in \mathbb{R}^{n_i}, \|\mathbf{y}_i\|=1} \mathbf{y}_i^\intercal A_{\dot{G}|_{D_i}} \mathbf{y}_i,$$

where $\mathbf{y}_i$ is the restriction of $\mathbf{x}_i$ on vertices of $\dot{G}|_{D_i}$. Again, by the Rayleigh principle, the right hand side of the previous equality is the index of $\dot{G}|_{D_i}$, $\lambda_1(\dot{G}|_{D_i})$. Together with the inequality of (1), this leads to $\lambda_1(\dot{G}) \leq \sum_{i=1}^{\ell} \lambda_1(\dot{G}|_{D_i})$. Since D and $\{D_1, D_2, \ldots, D_\ell\}$ were arbitrary, the desired result follows. □

Corollary 2.3. *For every edge-decomposition* $\{E_1, E_2, \ldots, E_k\}$ *of a signed graph* $\dot{G}$,

$$\lambda_1(\dot{G}) \leq \sum_{i=1}^{k} \lambda_1(\dot{G}|_{E_i}).$$

Proof. This result follows from Proposition 2.2 since for every partition $\{D_1, D_2, \ldots, D_\ell\}$ of $\{E_1, E_2, \ldots, E_k\}$, the collection $\{\dot{G}|_{D_1}, \dot{G}|_{D_2}, \ldots, \dot{G}|_{D_\ell}\}$ is contained in $\{\dot{G}|_{E_1}, \dot{G}|_{E_2}, \ldots, \dot{G}|_{E_k}\}$. □

Example 2.4. In this example the edge-decomposition of the graph $\dot{G}$ of Example 2.1 is used to illustrate the difference between the results of the previous two statements. Accordingly, for the decomposition D (of the previous example), Corollary 2.3 gives

$$\lambda_1(\dot{G}) \leq \lambda_1(\dot{C}_6) + \lambda_1(C_5) + \lambda_1(P_4) = \sqrt{3} + 2 + \frac{1+\sqrt{5}}{2} \approx 5.3501.$$

For the partition $\{E(\dot{C}_6) \cup E(C_5), E(P_4)\}$, Proposition 2.2 gives

$$\lambda_1(\dot{G}) \leq \lambda_1(\dot{C}_6 \cup C_5) + \lambda_1(P_4) = 2 + \frac{1+\sqrt{5}}{2} \approx 3.6180.$$

In other words, in Corollary 2.3, whenever the subgraphs induced by particular sets of edges, say E_r and E_s, are vertex disjoint, then $\min\{\lambda_1(\dot{G}|_{E_r}), \lambda_1(\dot{G}|_{E_s})\}$ can be dropped from the right hand side of the corresponding inequality.

If every edge subset E_i induces a cycle, then the corresponding decomposition is called a cycle decomposition of $\dot{G}$.

Proposition 2.5. *A signed graph has a cycle decomposition if and only if it is Eulerian.*

Proof. First, if a signed graph, say $\dot{G}$, is Eulerian, then a cycle decomposition is obtained by extracting cycles from an Eulerian trail. For the converse, let u be a vertex of $\dot{G}$. Then every cycle of a decomposition either passes through u in which case it takes exactly two edges incident with u, or does not pass through u in which case it does not take any edge incident with u. Accordingly, u has an even degree, which implies that $\dot{G}$ is Eulerian. □

If $\dot{G}$ is Eulerian, then there is the following consequence.

Proposition 2.6. *If* $\dot{C}_{j_1}, \dot{C}_{j_2}, \ldots, \dot{C}_{j_k}$ *form a cycle decomposition of a signed graph* $\dot{G}$*, then*

$$\lambda_1(\dot{G}) \leq 2 \sum_{i=1}^{k} c_i, \tag{2}$$

where

$$c_i = \begin{cases} 1 & \text{if } \dot{C}_{j_i} \text{ is positive}, \\ \cos\dfrac{\pi}{j_i} & \text{otherwise}. \end{cases}$$

Equality is attained if $\dot{G}$ *is a signed cycle or is balanced and* $\dot{C}_{j_i}$ *is a Hamiltonian cycle for every i.*

Proof. Corollary 2.3 gives $\lambda_1(\dot{G}) \leq \sum_{i=1}^{k} \lambda_1(\dot{C}_{j_i})$. On the other hand, Theorem 4.1 of (Simić and Stanić, 2016) implies that $\lambda_1(\dot{C}_{j_i})$ is either 2 or $2\cos\frac{\pi}{j_i}$ (depending on whether $\dot{C}_{j_i}$ is positive or not), which leads to the desired inequality.

If $\dot{G}$ is a signed cycle, then it has a unique cycle decomposition consisting of itself, and the equality in (2) follows. If the cycles are Hamiltonian, then $\dot{G}$ is necessarily regular, say of vertex degree r. In this case, $k = \frac{r}{2}$ and $c_i = 1$ for every i (since $\dot{G}$ is balanced), which gives $r = \lambda_1(G) = \lambda_1(\dot{G}) \leq 2\sum_{i=1}^{r/2} c_i = r$, and the proof is completed. □

The next proposition, together with the subsequent text, considers the quality of the bound obtained in the previous result in case of regular signed graphs.

Proposition 2.7. *Let* $\dot{C}_{j_1}, \dot{C}_{j_2}, \ldots, \dot{C}_{j_k}$ *be a cycle decomposition of a regular signed graph with n vertices and even vertex degree r. Then*

$$2\sum_{i=1}^{k} c_i \geq r \cos\frac{\pi}{n}, \tag{3}$$

where the parameters c_i are defined in Proposition 2.6. The equality holds if and only if $\dot{C}_{j_i}$ is a negative Hamiltonian cycle for every i.

Proof. It holds

$$2\sum_{i=1}^{k} c_i \geq 2\sum_{i=1}^{k} \cos\frac{\pi}{j_i}, \tag{4}$$

with equality if and only if C_{j_i} is negative for every i.

Since $\sum_{i=1}^{k} j_i = \frac{m}{n}$, $\frac{m}{2k}$ is the average length of cycles in the decomposition. Without loss of generality, assume that $j_1 \leq j_2 \leq \cdots \leq j_k$.

For $j_1 = j_k$,

$$2\sum_{i=1}^{k} \cos\frac{\pi}{j_i} = 2\sum_{i=1}^{k} \cos\frac{2k\pi}{rn}. \tag{5}$$

Since $\cos\frac{\pi}{x} > \cos\frac{\pi}{x+1}$ for $x \geq 3$ (and $\frac{m}{2k} \geq 3$), it holds

$$2\sum_{i=1}^{k} \cos\frac{2k\pi}{rn} \geq r \cos\frac{\pi}{n}, \tag{6}$$

with equality if and only if $k = \frac{r}{2}$. Now, (5) and (6) lead to (3).

For $j_1 = j_k - 1$,

$$2\sum_{i=1}^{k} \cos\frac{\pi}{j_i} = 2\left(s \cos\frac{\pi}{\lfloor\frac{m}{2k}\rfloor} + t \cos\frac{\pi}{\lceil\frac{m}{2k}\rceil}\right), \tag{7}$$

with $s\lfloor\frac{m}{2k}\rfloor + t\lceil\frac{m}{2k}\rceil = \frac{m}{2}$.

For $j_1 < j_k - 1$, an easy trigonometric calculus gives $\cos\frac{\pi}{j_1} + \cos\frac{\pi}{j_k} > \cos\frac{\pi}{j_1+1} + \cos\frac{\pi}{j_k-1}$. Accordingly,

$$2\sum_{i=1}^{k} \cos\frac{\pi}{j_i} > 2\left(\left(\sum_{i=2}^{k-1} \cos\frac{\pi}{j_i}\right) + \cos\frac{\pi}{j_1+1} + \cos\frac{\pi}{j_k-1}\right).$$

In other words, j_1 and j_k may be replaced with $j_1 + 1$ and $j_k - 1$ to decrease the sum on the left hand side. Now, one may repeat the following procedure until $j_1 \leq j_k - 1$: relabel the parameters $j_1 + 1, j_2, \ldots, j_{k-1}, j_k + 1$ into $j_1 \leq j_2 \leq \cdots \leq j_k$ and apply the previous replacing. In the end,

$$2\sum_{i=1}^{k} \cos\frac{\pi}{j_i} > 2\left(s \cos\frac{\pi}{\lfloor\frac{m}{2k}\rfloor} + t \cos\frac{\pi}{\lceil\frac{m}{2k}\rceil}\right), \tag{8}$$

with $s\lfloor\frac{m}{2k}\rfloor + t\lceil\frac{m}{2k}\rceil = \frac{m}{2}$, as before.

The argument stated before the inequality (6) implies

$$2\left(s \cos\frac{\pi}{\lfloor\frac{m}{2k}\rfloor} + t \cos\frac{\pi}{\lceil\frac{m}{2k}\rceil}\right) \geq r \cos\frac{\pi}{n}, \tag{9}$$

again with equality if and only if $k = \frac{r}{2}$. The inequality (3) follows by taking (4), (7) and (9) for $j_1 = j_k - 1$, and (4), (8) and (9) for $j_1 < j_k - 1$.

Consider now the case of equality in (3). If C_{j_i} is a negative Hamiltonian cycle for every i, then $c_i = 2\cos\frac{\pi}{n}$ and there are $\frac{r}{2}$ cycles in the decomposition, and the equality follows. Assume now that the equality holds. This, in the first place, implies equality in (4), which means that all cycles are negative.

For $j_1 = j_k$, there is equality in (6), which implies $k = \frac{r}{2}$, and thus the cycles are Hamiltonian.

For $j_1 = j_k - 1$, it holds $j_1 = \lfloor\frac{m}{2k}\rfloor < \lceil\frac{m}{2k}\rceil = j_k$, which in particular means that $k \neq \frac{r}{2}$. The latter gives the strict inequality in (9), which contradicts the assumption on equality in (3).

For $j_1 < j_k - 1$, the strict inequality in (8) contradicts the same assumption, and the proof is completed. □

The previous proposition gives a lower amplitude for the upper bound of (2). Moreover, the right hand side of (3) is always less than r (where r is the vertex degree and simultaneously the upper bound for $\lambda_1(\dot{G})$). In addition, if $\dot{G}$ is unbalanced and allows a Hamiltonian decomposition, then the upper bound of (2) is non-trivial in the sense that it is less than r. This also applies to the question of how the largest eigenvalue of a signed graph differs from the index of its underlying graph. Namely, if $\dot{G}$ allows a Hamiltonian decomposition, then $\lambda_1(G) - \lambda_1(\dot{G}) \geq r - \sum_{i=1}^{r/2} c_i$. Finally, it is worth mentioning that Hamiltonian decompositions of graphs have attracted a notable attention. First, such a (signed) graph must be regular of even vertex degree. Next, testing whether an arbitrary graph has a Hamiltonian decomposition is *NP*-complete (Péroche, 1984). It is known that every complete graph of odd order has a Hamiltonian decomposition (Hanfried and Ringel, 1991). Random regular graphs of even degree almost always have a Hamiltonian decomposition (Kim and Wormald, 2001). For more results, the reader is referred to (Bermond, 1978; Kotzig, 1957; Martin, 1976) and references therein.

According to (Bryant et al., 2014), every complete graph of even order allows a decomposition consisting of a perfect matching and the set of Hamiltonian cycles. Together with Propositions 2.2 and 2.6, this gives the following consequence; the proof follows immediately.

Corollary 2.8. *For a complete signed graph $\dot{G}$ of even order n,*

$$\lambda_1(\dot{G}) \leq 2\sum_{i=1}^{(n-2)/2} \lambda_1(\dot{C}_n^i) + 1,$$

where $\dot{C}_n^1, \dot{C}_n^2, \ldots, \dot{C}_n^{(n-2)/2}$ are edge-disjoint Hamiltonian cycles of $\dot{G}$.

In the particular case of unsigned graphs, there is the following result based on the recently proved Barát-Thomassen conjecture (Barát and Thomassen, 2006). The conjecture states that for any tree T with m edges, there exists an integer $k = k(T)$ such that every k-edge-connected graph whose number of edges is divisible by m has a T-decomposition, that is an edge-decomposition such that all the corresponding subgraphs are isomorphic to T. The conjecture was confirmed to be true in (Bensmail et al., 2017). Accordingly, there is the following result.

Corollary 2.9. *For a tree T, if a graph G has a T-decomposition and the corresponding subgraphs are partitioned into s classes in such a way that two with a common vertex are not in the same class, then $\lambda_1(G) \leq s\lambda_1(T)$.*

Proof. This result is obtained by setting $D_i = E(T)$ for every i in Proposition 2.2. □

Of course, the previous corollary remains valid for signed graphs $\dot{G}$ as $\lambda_1(\dot{G}) \leq \lambda_1(G)$ and every signed tree switches to its underlying tree.

3. Conclusion

The most frequently investigated eigenvalue of a graph (weighted graph, directed graph, or any other generalization) is the largest eigenvalue of its adjacency matrix. It is usually estimated by the upper or the lower bounds expressed in terms of structural parameters and/or the eigenvalues of related graphs. Many details can be found in (Stanić, 2015). This paper deals with the concept of signed graphs that encapsulates the concept of ordinary graphs in the sense that every graph is interpreted as a particular signed graph. The obtained results offer upper bounds for the largest eigenvalue of a signed graph in terms of the largest eigenvalues of subgraphs induced by edge-decompositions. A particular attention is devoted to the so-called cycle decompositions of regular signed graphs and Hamiltonian signed graphs. As mentioned in the previous section, cycle decompositions (in particular, Hamiltonian decompositions) have received a great deal of attention over the last seven decades, and the results of this paper are related to those obtained in the corresponding references.

Needless to add, every result remains valid for ordinary graphs. In addition, if a signed graph $\dot{G}$ is regular of vertex degree r, then the least eigenvalue of its Laplacian matrix (the matrix $D_{\dot{G}} - A_{\dot{G}}$, where $D_{\dot{G}}$ is the diagonal matrix of vertex degrees) is $r - \lambda_1(\dot{G})$, which means that the results dealing with regular signed graphs can be formulated in terms of this eigenvalue, as well.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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